Calculus 1
$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

Worksheet: Optimization (coordinate plane)

Name_____
Block _____ Date_____

1) What are the dimensions of the largest rectangle that can fit between the curve of $f(x) = \sqrt{25-x^2}$ and the x-axis?
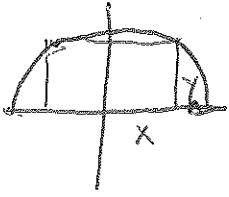
$A = x \cdot y$

$A = X \cdot \sqrt{25-x^2} = X(25-x^2)^{1/2}$

$A' = 1 \cdot \sqrt{25-x^2} + X \cdot \frac{1}{2}(25-x^2)^{-1/2}(-2x)$

$A' = \sqrt{25-x^2} - X^2(25-x^2)^{-1/2} = 0$

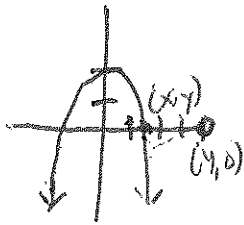$\sqrt{25-x^2} = x^2/\sqrt{25-x^2}$

$25-x^2 = x^2$

$25 = 2x^2$

$12.5 = x^2$

$X = 3.536$
$Y = 3.536$

$\boxed{7.071 \times 3.536}$

2) Find the point on the graph of $y = -x^2 + 2$ that is closest to the point $(4,0)$.

$d = \sqrt{(x-4)^2 + (y-0)^2}$

$d = \sqrt{(x-4)^2 + (-x^2+2)^2}$

$d = \sqrt{x^2 - 8x + 16 + x^4 - 4x^2 + 4}$

$d = \sqrt{x^4 - 3x^2 - 8x + 20}$

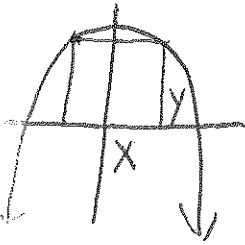$d' = \frac{1}{2}(x^4 - 3x^2 - 8x + 20)^{-1/2}(4x^3 - 6x - 8) = 0$

$X = 1.647$
$Y = -.713$

$\boxed{(1.647, -.713)}$

3) What are the dimensions of the largest rectangle that can fit between the curve $f(x) = 12 - x^2$ and the x-axis?
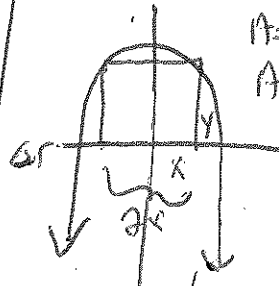
$A = x \cdot y$

$A = X(12-x^2)$

$A = 12x - x^3$

$A' = 12 - 3x^2$

$12 = 3x^2$

$4 = x^2$

$X = \pm 2$

$\boxed{4 \times 8}$

$A = 2x \cdot y$

$A = 2x(10-x^2)$

$A = 20x - 2x^3$

$A' = 20 - 6x^2 = 0$

$6x^2 = 20$

$x^2 = \frac{20}{6}$

$X = 1.2$

$\boxed{(1.444, 3.130)}$

4) What point on $g(x) = \sqrt[3]{x} + 2$ is closest to the point $(2,1)$?
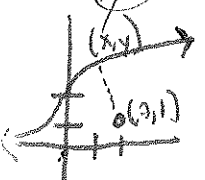
$d = \sqrt{(x-2)^2 + (y-1)^2}$

$d = \sqrt{(x-2)^2 + (\sqrt[3]{x}+2-1)^2}$

$d = \sqrt{(x-2)^2 + (\sqrt[3]{x}+1)^2}$

$d = \sqrt{x^2 - 4x + 4 + \sqrt[3]{x^2} + 2\sqrt[3]{x} + 1}$

$d = \sqrt{x^2 - 4x + x^{2/3} + 2x^{1/3} + 1}$

$d = \sqrt{x^2 - 4x + x^{2/3} + 2x^{1/3} + 5}$

$d' = \frac{1}{2}(x^2 - 4x + x^{2/3} + 2x^{1/3} + 5)(2x - 4 + \frac{2}{3}x^{-1/3} + \frac{2}{3}x^{-2/3}) = 0$

$X(.14)$ 1.444

$(.14, .519)$
$(1.444, 3.130) = 2.201$

5) What are the dimensions of the largest rectangle than can fit between the curve $f(x) = -x^3 + 27$, the x-axis, and the y-axis?
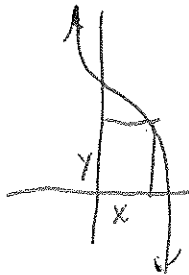
$A = x \cdot y$

$A = X(-x^3 + 27)$

$A = -x^4 + 27x$

$A' = -4x^3 + 27 = 0$

$4x^3 = 27$

$x^3 = \frac{27}{4}$

$X = 1.890$

$y = -(1.89)^3 + 27$

$Y = 20.249$

$\boxed{1.890 \times 20.249}$